Multivariable Calculus Common Topics List[1]

1. **Multivariable Functions**
2. **3-D space**
   (a) Distance
   (b) Equations of planes, spheres, etc.
   (c) Two-variable function graphs
   (d) Sections, level curves, and contour diagrams
3. **Vectors**
   (a) Arithmetic on vectors, graphically and by components
   (b) Dot Product and projection
   (c) Cross Product
4. **Limits are more complicated than in the one-variable case**
5. **Partial Derivatives**
   (a) Compute using the definition of partial derivatives
   (b) Compute using differentiation rules
   (c) Approximate given a contour diagram or other info about a function
   (d) Estimate signs from real-world desciption
   (e) Find the tangent plane
   (f) Compute higher-order partials
   (g) Mixed partials are equal under certain conditions
6. **Directional derivatives**
   (a) Estimate from contour diagram
   (b) Compute using limit definition
   (c) Compute using dot product with the gradient
7. **Gradient**
   (a) Compute the gradient
   (b) Points in the direction of fastest increase
   (c) Length is the directional derivative in that direction
   (d) Perpendicular to level set
   (e) Draw gradient vector given a contour diagram

[1]This list was approved by the department on 4/10/19

8. **Curl and Divergence**
9. **Chain Rule**
10. **Optimization**
    (a) Locate and classify critical points in a contour diagram
    (b) Find critical points given a formula
    (c) Find maxima and minima
    (d) Second derivative test
    (e) Extreme value theorem, including understanding of closed and bounded
    (f) Lagrange multipliers
11. **Integration**
    (a) Predict the sign of a multiple integral
    (b) Compute a multiple integral
    (c) Sketch region of integration
    (d) Choose or change the order of integration
    (e) Polar and cylindrical coordinates
12. **Parametrized Curves**
    (a) Construct parametrizations of lines, circles, and explicitly defined curves
    (b) Velocity and speed
13. **Vector Fields**
    (a) Sketch a vector field with a given formula
    (b) Recognize a conservative (gradient) vector field
    (c) Find a formula for a potential function of a vector field
14. **Line Integrals**
    (a) Given a picture of a vector field, predict the sign of a line integral
    (b) Compute a line integral using explicit parametrization formula
    (c) Compute arc length of a curve
    (d) For a gradient field, compute using the fundamental theorem of line integrals
    (e) For a gradient field, compute using a reparametrization and path independence
    (f) For a gradient field, the line integral over a loop is zero
15. **Green's Theorem**

Suggested Additional Topics

- Paraboloids, hyperboloids, and ellipsoids
- Spherical Coordinates
- Find plane tangent to implicit surface by viewing the surface as a level set
- Implicit differentiation
- Jacobian formula for reparametrizing a multiple integral into a different coordinate system
- Surface Integrals over planes, spheres, cylinders
- Stokes's Theorem
- Divergence Theorem

Optional Topics

- Level Surfaces
- ϵ-δ limit proofs
- Differentiability is more complicated than the one-variable case
- Optimization on the boundary of a region by substitution
- Jacobian formula for surface integral over an arbitrary surface
- Proofs using vectors
- Signs of second partials from contour diagram
- Use limits to discuss existence of global maxima and minima
- Intersections of a curve with a surface
- Collisions and intersections of parametrized curves
- Distances involving parametrized lines
- Parametrize a complicated curve by summing parametrizations of simple component motions
- Give a plausible formula for a vector field from a sketch