

Calculus I (Math 110, Math 114) Topics List ¹

(*) means that this topic should be included in Math 114.

(**) means that this is an optional topic in either Math 110 or Math 114.

0. Functions of Pre-calculus ()**

(These topics are pre-requisite, therefore optional, but can be reviewed)

- (a) Cartesian coordinate systems
- (b) Real-valued functions: domain, co-domain, range.
- (c) One-to-one functions.(**)
- (d) Algebra and transformation of functions. Composition, inverse functions.
- (e) Linear functions, quadratic functions, powers, polynomials, rational functions, exponential functions, hyperbolic functions, logarithmic functions, piecewise functions, absolute value function, step functions.
- (f) Graphing of all the familiar functions above.
- (g) Properties of functions: Even, odd, increasing/decreasing, roots.
- (h) Exponential and logarithmic properties and identities.
- (i) Trigonometric functions, their properties, and their identities.
- (j) Conic sections; parabola, hyperbola, ellipse.
- (k) Inequalities, graphing their regions.
- (l) Parametrized curves: $r(t)=(x(t), y(t))$.
- (m) The polar coordinate system, polar curves.

1. Limits and Continuity

- (a) Intuitive definition of a limit at a point with graphs and tables.
- (b) Right-hand and left-hand limits at a point.
- (c) Epsilon-delta definition of the limit. (*)
- (d) Limit laws.
- (e) The Squeeze Theorem.
- (f) Limits of polynomials, rational functions, and algebraic functions with analytic techniques dealing with $0/0$.
- (g) Trigonometric limits: $\sin(x) / x$, $\sin(x) / x^2$, $(1 - \cos(x)) / x$, etc.

¹This list was approved by the department on 10/25/22.

- (h) Infinite limits and limits at infinity; Vertical and horizontal asymptotes, removable singularities.
- (i) Definition of continuity at a point.
- (j) Permanence of continuity; linear combinations, products, quotients, composition.
- (k) Continuity on a domain.
- (l) Left and right continuity; continuity on a closed and bounded interval.
- (m) The Intermediate Value Theorem with examples.

2. The Derivative

- (a) The geometric intuition of a tangent line.
- (b) Differentiability at a point.
- (c) Differentiability on an interval.
- (d) Finding the equation of a tangent line to a graph at a point.
- (e) Parametric equations and their derivatives.
- (f) Implicitly defined curves and Implicit Differentiation.
- (g) Rules of differentiation; linear rule, power rule (proof*), product rule (proof*), quotient rule, chain rule.
- (h) Higher order derivatives.
- (i) The Inverse Function Theorem; (**)
- (j) Relation between derivatives of inverse function pairs.
- (k) Derivatives of exponential, logarithm, inverse trig functions.
- (l) Derivatives of hyperbolic functions. (*)
- (m) Logarithmic differentiation.
- (n) Rolle's Theorem and The Mean Value Theorem with examples. (*)
- (o) Relative and absolute extrema. Fermat's theorem, critical numbers. First and second derivative tests. Concavity. Inflection points.
- (p) The Extreme Value Theorem with examples.

3. Applications of Differentiation

- (a) Indeterminate forms (l'Hopital's rule).
- (b) Limit definition of e^a . (**)
- (c) Curve sketching.
- (d) Optimization.

- (e) Related rates.
- (f) Linear approximations.
- (g) Differentials. (**)
- (h) Exponential growth and decay.
- (i) Approximation by Taylor polynomials. (*)
- (j) Elasticity. (**)
- (k) Newton's method. (*)
- (l) Velocity, acceleration, kinematics.
- (m) Partial derivatives. (**)
- (n) Verifying that a function satisfies a differential equation. (*)
- (o) Use of Technology.

4. Riemann Sums & Integration

- (a) Antiderivatives.
- (b) Riemann Sums, left, right, midpoint, tagged.
- (c) Geometric interpretation; areas under graphs.
- (d) The "Calculus" definition of the (definite) Riemann integral
- (e) Properties of the definite integral, comparative inequalities.
- (f) The Fundamental Theorem of Calculus
- (g) Computing definite integrals. (**)